Application of Info-Gap Decision Theory to Assess the Effect of Parametric Uncertainty in a Thermal Conductivity Model of Uranium Oxide Light Water Nuclear Reactor Fuel

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Abstract

In a companion paper [Unal, Stull, Williams, 2013], the parametric uncertainty in a thermal conductivity model of uranium oxide fuel was assessed using Markov Chain Monte Carlo (MCMC) analysis. This paper presents a similar assessment using an independent methodology aimed at examining the predictive capability of the empirically-derived thermal conductivity model adopted in [Unal, Stull, Williams, 2013]. The methodology considers the effect of uncertain parameters – a plausible reality in the context of empirically-derived models – on the ability of the model to predict uranium dioxide fuel conductivity data from open literature sources. The results obtained from this assessment are similar to the results found in the companion paper that led the authors to question the predictive capability of the FRAPCON model for predicting the thermal conductivities associated with *irradiated* fuel samples.

Keywords

Nuclear; Info-Gap Decision Theory; Reactor; Parametric; Uncertainty; Thermal Conduction; Uranium Dioxide

Introduction

Given the increased reliance on modeling and simulation to make predictions (both interpolatory and extrapolatory) in the absence of experimental data, it befits the scientific community to explore and report upon uncertainty quantification techniques applied to previously adopted (or at least well-accepted) empirical models that derive from, or pertain to, substantial experimental data sets. Such studies, while more applied in nature, provide guidance for conducting studies on codes that are still under development.

The ceramic nuclear fuel, UO_2, used in light water nuclear reactors is known to be a very poor semiconductor. The available thermal conductivity models are summarized in the companion paper [Unal, Stull, Williams, 2013]. For brevity, we will not repeat the summaries here, except to provide Equations (1) and (2) for the fresh and irradiated fuel samples, respectively.

$$K_{95} = \frac{1}{A+BT} + \frac{E}{T^2}\exp\left(-\frac{F}{T}\right). \quad (1)$$

$$K_{95} = \frac{1}{A+BT+f(Bu)+(1-0.9\exp(-0.04Bu))g(Bu)h(T)} + \frac{E}{T^2}\exp(-\frac{F}{T}) \quad (2)$$

In this paper, we will consider Equations (1) and (2) and attempt to understand the correlation issues between A and B from Equation (1) and f, g, h, and the extra term from Equation (2).

Other methodologies are applied to the same problem for comparison. The use of MCMC analysis has been discussed in the companion paper; a genetic algorithm optimization was also used but not discussed in this paper. While much insight was gained through the genetic algorithm optimization as to the behavior of the models with respect to the available data, the primary conclusion was that the thermal conductivities associated with the fresh fuel samples could not be accurately predicted when employing the data/model from the irradiated fuel samples.

This paper focuses on using a non-probabilistic approach, namely info-gap decision theory, to assess the "robustness" of the FRAPCON model, when the

uncertain parameters are considered to be truly unknown quantities, centered about their nominal values.

Info-Gap Decision Theory Analyses

The assessment technique highlighted in this paper adopts a framework anchored in *Info-Gap Decision Theory* (IGDT). Simply stated, IGDT offers a theoretical framework that aims to facilitate decision-making in the face of severe uncertainty [Ben-Haim, 2006]. A typical IGDT analysis aims to quantify the "info-gap robustness" of a series of decisions, ultimately providing the decision-maker with a series of so-called "robustness curves." These robustness curves illustrate, for each decision, the relationship (or trade-off) between the *performance* of a decision and the *robustness* against the uncertainty that impacts the decision. Rounding out the analysis, the decision-maker defines a minimum performance requirement (*e.g.* maximum error) that then imposes a ranking of the decisions according to how robustly they meet that performance requirement.

For the purposes of this paper, IGDT will be employed in a manner that is more analogous to a sensitivity analysis than a traditional info-gap analysis. However, as pointed out in [Stull et al., 2012], "the advantage of estimating sensitivities by way of an IGDT analysis is that we do not rely on approximating local or global derivatives or adjoints; IGDT produces sensitivities that are free of these limiting assumptions." To this end, the following discussion serves to provide a more detailed overview of IGDT, where the authors are quick to note that much of the following discussion has already been presented in [Stull et al., 2012]. This discussion is necessarily more thorough than the discussions provided below, given the authors' perception that IGDT is a relatively new theory, particularly to the UQ community. Moreover, the authors will make every effort to maintain the general treatment of IGDT, as such a treatment will facilitate the use of IGDT to solve a broader class of problems faced by UQ researchers. Interested readers are pointed toward [Ben-Haim, 2006] for a complete treatment of IGDT as well as the references contained in [Ben-Haim, 2012] for examples of how IGDT may be applied to various problems and problem types.

Any IGDT analysis begins with three central questions: (1) "What is the decision that needs to be made?" (2) "Where is the uncertainty that affects the quality of this decision?" (3) "How is the quality (or performance) of this decision quantified?" In this study, the decision we wish to inform through the IGDT analysis is whether or not the FRAPCON model remains predictive when its empirically-derived parameters are allowed to vary. Note that the previous sentence has also answered the second question. The mechanism by which IGDT represents uncertainty is formally referred to as an "info-gap model of uncertainty." This representation is not necessarily a probabilistic model, as might be implied by the term "uncertainty." IGDT asserts that uncertainty is simply represented as a gap in information, about which little is assumed other than how it relates to a "nominal model" of the system. Often, these nominal models are derived from analyzing experimental data, eliciting expert opinion, compiling consistent theories, or they may simply be educated guesses offered by the analyst. For the results reported upon herein, this nominal model corresponds to the FRAPCON model, as provided in [Geelhood et al., 2011]. Regardless of how the nominal model is defined, however, it is important for the reader to keep in mind that such a model represents the situation in which <u>no</u> uncertainty is expected in the model parameters (a precariously optimistic view of the world, in the authors' opinions).

Answering the third and most subjective of the three questions, we define a performance function, consistent with the objective function employed for the optimization-based assessments reported elsewhere:

$$E_{\mathrm{RMS}}(\boldsymbol{\theta}) = \sum_{i=1}^{n} \left(\frac{1}{m_i} \sum_{j=1}^{m_i} \left(K_{i,j}^{\exp} - K_{i,j}^{\mathrm{mod}}(\boldsymbol{\theta}) \right)^2 \right)^{\frac{1}{2}}. \quad (3)$$

n corresponds to the number of data sets available for the analysis, and m_i corresponds to the number of data points available within the ith data set. Note that while a measure of accuracy is being employed in this study, this need not always be the case for IGDT analyses. Generally speaking, performance could be defined as a response feature that is not to exceed a certain threshold value (*e.g.* the deflection of a cantilevered beam) or as being related to efficiency of a decision (*e.g.* the time required to exercise a computer code or code segment).

Therefore, the IGDT analysis contained herein addresses the question: "What is the permissible level of ignorance about the parameters comprising the FRAPCON model that can be tolerated, which does

not cause the predictions to exceed a maximum allowable level of error." This question is obviously of considerable importance when asked within the context of model calibration, where calibrations are often done without regard to what the parameter variations mean, leading to models that fit the experimental data but may exhibit little-to-no predictive capability. The following section provides a more formal treatment of the IGDT problem formulation.

Problem Formulation

The IGDT problem formulation begins generally, by considering a set of analytical models $\boldsymbol{\mathcal{M}}$, all of which describe the thermal conductivity of fresh and/or irradiated UO_2 fuel pellets. From $\boldsymbol{\mathcal{M}}$ are chosen M models $m_I(\boldsymbol{\theta}^{m_I}) \forall I = 1,2,\ldots,M$, where $\boldsymbol{\theta}^{m_I}$ is a column vector that collects the parameters for the m_Ith model, and $\widetilde{\boldsymbol{\theta}}^{m_I}$ denotes the nominal model parameters associated with the m_Ith model. Note that the length of the vectors $\boldsymbol{\theta}^{m_I}$ need not be consistent across the M analytical models.

Having provided the above definitions, the info-gap model of uncertainty may be defined as

$$\boldsymbol{\mathcal{U}}(m_I, \alpha) = \left\{ \boldsymbol{\theta}^{m_I} : \left| \frac{\boldsymbol{\theta}_j^{m_I} - \widetilde{\boldsymbol{\theta}}_j^{m_I}}{\widetilde{\boldsymbol{\theta}}_j^{m_I}} \right| \leq \alpha, \forall j \right\}, \alpha \geq 0. \quad (4)$$

Equation (4) is representative of a class of info-gap models referred to as "envelope-bound" (or "error-bound") info-gap models [Ben-Haim, 2006], and $\boldsymbol{\mathcal{U}}(m_I, \alpha)$ represents a *nested set* of m_I-type analytical models, whose parameters may assume values within the bounded interval defined by α: an unknown scalar quantity, referred to as the "horizon-of-uncertainty." This simply means that the extent to which the model parameters can deviate from their nominal values is, in actuality, unknown (though practically speaking, decision analysts will often set limits on α). Put another way, Equation (4) explicitly defines *sets* of model parameters from which the analyst may choose, given a fixed level of the horizon-of-uncertainty (or level of lack of knowledge). As the level of lack of knowledge increases, the set of potential parameter values also grows.

As a further aid to the reader, a representative illustration of an envelope-bound info-gap model is presented in Fig. 1 where it is seen that at $\alpha = \alpha_2$, the range of values that the parameter labeled b_i^{u} may assume also includes that defined at $\alpha = \alpha_1$. Likewise, the range of values that b_i^{u} may assume at $\alpha = \alpha_3$, includes that defined at $\alpha = \alpha_2$. Note that the illustration does not imply that an info-gap model of uncertainty is limited to nested intervals. The parameter b_i^{u} could, for example, represent the standard deviation or entropy of a family of probability laws that describe variability. With the info-gap model of uncertainty (*i.e.* Equation (4)) and the performance function (*i.e.* Equation (3)) defined, the central concepts of IGDT, namely "robustness" and "opportunity," may be introduced. Simply stated, robustness is a reflection of the immunity of the model(s) to uncertainty in the model form or its parameters [Ben-Haim, 2006]. In the present context, highly robust models will have two characteristics: (a) acceptable agreement between the nominal response features predicted by $m_I(\widetilde{\boldsymbol{\theta}}^{m_I})$ and the experimental data and (b) little change in the evaluation of Equation (3) over the set defined by Equation (4). Conversely, analytical models with low levels of robustness violate one or both of the above characteristics. Clearly, higher robustness is better than lower robustness at a similar level of performance; this observation offers the mechanism for model selection.

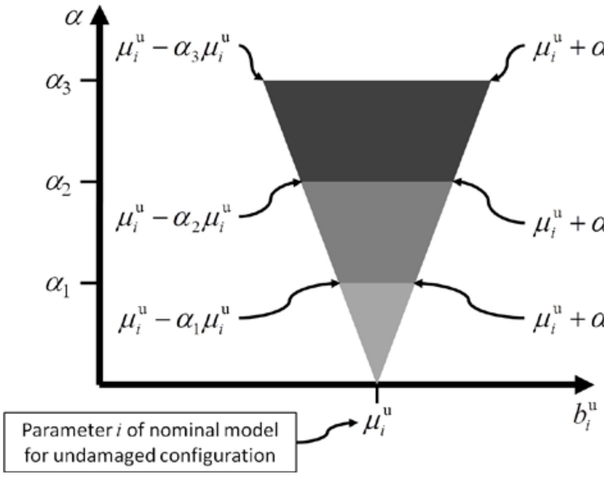

FIG. 1 REPRESENTATIVE ILLUSTRATION OF THE NESTED NATURE OF EQUATION (4)

Opportunity then relates to the potential to *improve* the performance of a model, given uncertainty in the functional form or parameters that define the model. Considering the problem of model calibration, the nominal model may be thought of as an initial, best guess at the values of the parameters. Subsequently, calibration will lead to "improved" definitions of these parameters such that the model predictions better fit the available experimental data. Opportunity would uncover this potential, but it should not be considered

as analogous to calibration; the intent of IGDT is only to inform the decision-making process.

The robustness and opportunity functions are formally stated as

$$\hat{\alpha}(m_I, E_c) = \max_{\alpha}\{\max_{\boldsymbol{\theta}^{m_I}\in\mathcal{U}(m_I,\alpha)}\{E_{\mathrm{RMS}}(\boldsymbol{\theta})\} \le E_c\} \quad (5)$$

and

$$\hat{\beta}(m_I, E_c) = \max_{\alpha}\{\min_{\boldsymbol{\theta}^{m_I}\in\mathcal{U}(m_I,\alpha)}\{E_{\mathrm{RMS}}(\boldsymbol{\theta})\} \ge E_c\}, \quad (6)$$

respectively. Equation (5) that defines the robustness function has two parts: an inner maximum and an outer maximum. The inner maximum reflects the worst-case performance of the family of models defined by $m_I(\boldsymbol{\theta}^{m_I})$, where the parameters $\boldsymbol{\theta}^{m_I}$ are selected from the set $\mathcal{U}(m_I, \alpha)$. Even though the robustness function identifies the worst-case performance within the family of models, the inequality of Equation (5) guarantees that this worst fidelity to data does not exceed the level E_c of prediction error. Therefore, E_c, which is a threshold selected by the analyst, can be understood as a requirement for prediction accuracy. Incorporating this inner maximum, the outer maximum is then interpreted as seeking the maximum value of the horizon-of-uncertainty α, for which the worst-case performance of the analytical model does not exceed a critical level E_c.

While this knowledge is useful, in a practical application, it can be of far more benefit to the decision analyst to plot the inner maximum for various values of the horizon-of-uncertainty α, producing the aforementioned "robustness curve." Such curves then provide the decision analyst with insight as to how the worst-case performance of a family of models varies with increasing levels of lack of knowledge. In cases where multiple models are considered, the respective robustness curves may be overlaid allowing the decision analyst to weigh the merits of each model relative to others. An example of such a set of robustness curves is presented in Fig. 2. The black, dashed line of Fig. 2 follows the most robust model, starting with the model labeled m_3, then switching over to the model labeled m_2, as the horizon-of-uncertainty α increases.

Despite being a hypothetical example, Fig. 2 demonstrates several results that may arise from IGDT analyses. First is the idea that each of the five models demonstrates zero robustness when the nominal model parameters are assumed. This is analogous to an analyst employing his/her best guess at the model parameters, executing each model once, and treating the model exhibiting the least error as *the* answer to the problem. It is the authors' contention that such a stance represents a precariously optimistic view of the world, as epistemic uncertainty (*i.e.* a lack of *complete* knowledge about the behavior of the system) is present, but ignored when the model is analyzed only once. A reflection of this lack of knowledge is demonstrated in the non-zero errors produced by nominal models. That is, nominal models are incorrect, not necessarily because the analyst employed incorrect assumptions about the parameters, but possibly because the *form* of the model does not capture the full spectrum of system behavior.

The second result illustrated by Fig. 2 is that of model preference reversal. That is, as the level of uncertainty increases, the worst-case performances predicted by the different models do not change at the same rates, resulting in crossing of the robustness curves. It is noted that crossing does not always occur; robustness curves can follow parallel or divergent tracks such that crossing cannot occur. Examination of Fig. 2 reveals three levels of lack of knowledge where crossing of the robustness curves occurs. Faced with this situation, the decision analyst may choose the model that satisfies the minimum performance requirement, but admits the most lack of knowledge, as compared to the other models. This choice guarantees that predictions of the model meet the minimum performance requirement, even if some of the model parameters are selected erroneously. Alternatively, the decision analyst may assume a level of uncertainty and subsequently choose the model that offers the least error at that uncertainty. It may, however, be argued that the latter interpretation of the robustness curve is less aligned with the spirit of IGDT. An important message associated with the idea of model preference reversal is that the model producing the least prediction error may not be the best choice, if one is also uncertain about the functional form or parameters that comprise the model. That is, the so-called optimal model may become sub-optimal in the face of uncertainty, when compared against an alternative modeling strategy. This goes to the authors' earlier assertion that optimization-based calibration often did not provide a complete picture of the problem [Unal, Stull, Williams, 2013].

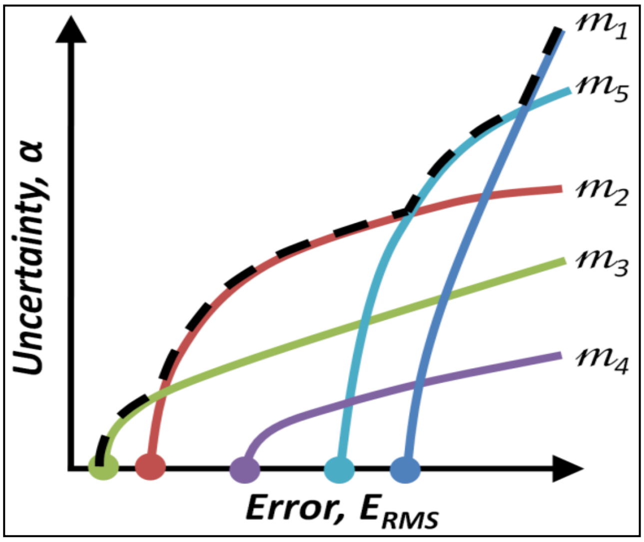

FIG. 2 ROBUSTNESS CURVES FOR A SET OF FIVE HYPOTHETICAL MODELS

Equation (6) presents the converse of the robustness function: the opportunity function. It is noted that the only difference between the Equations (5) and (6) is that the inner maximum in Equation (5) becomes an inner minimum in Equation (6). Translated into English, this inner minimum seeks the best-case performance of the analytical model defined by $m_I(\boldsymbol{\theta}^{m_I})$, where the parameters $\boldsymbol{\theta}^{m_I}$ are selected from the set $\boldsymbol{\mathcal{U}}(m_I, \alpha)$. This presents the situation where uncertainty about the parameters comprising a model can *improve* the performance of that model, as compared to the nominal condition, a situation referred to as "windfalling" [Ben-Haim, 2006]. Such knowledge could be used to inform calibration studies by quantifying the extent to which prediction accuracy could potentially be improved.

Ultimately, the question of whether to assess robustness or opportunity comes down to a question of whether the analyst is pessimistic or optimistic about his/her model. It is the authors' opinion that more often than not, analysts will wish to examine robustness. Consider, for example, a model that approximates a physical system known to have considerable implications with respect to life-safety. In this situation, analyzing opportunity would be of little-to-no use, given that few decision-makers (*e.g.* regulatory agencies, policy-makers) would formulate policies from knowledge of best-case scenarios. With that said, having knowledge of the opportunity of a decision (*e.g.* a model) at various levels of lack of knowledge can still present the analyst with valuable insight.

Robustness of Individual Model Parameters

The previous discussion has provided a reasonably complete introduction to info-gap decision theory, but as mentioned, the results discussed in this section are obtained from a less-than-traditional application of IGDT. This does not, however, imply that the previous discussion is superfluous. On the contrary, the IGDT framework as outlined above is applicable to any problem where uncertainty affects a decision aimed at selecting a model: an all too common situation in the scientific and engineering community. Moreover, in the absence of the context provided by the previous discussion, the following would have little meaning.

The "less-than-traditional application of IGDT" primarily refers to an extension of the framework outlined previously to compute robustness and opportunity functions that are specific to a single parameter. To accomplish this, the info-gap model in Equation (4) is modified such that all of the uncertain model parameters are held fixed at their nominal values, except for the k^{th} parameter. Equation (7) presents this modified info-gap model of uncertainty as

$$\boldsymbol{\mathcal{U}}_k(m_I, \alpha) = \left\{ \boldsymbol{\theta}^{m_I} : \left\{ \begin{matrix} \boldsymbol{\theta}_j^{m_I} = \tilde{\boldsymbol{\theta}}_j^{m_I}, \forall j \neq k \\ \left| \frac{\boldsymbol{\theta}_k^{m_I} - \tilde{\boldsymbol{\theta}}_k^{m_I}}{\tilde{\boldsymbol{\theta}}_k^{m_I}} \right| \leq \alpha \end{matrix} \right\} \right\}, \alpha \geq 0. \quad (7)$$

While this imposes additional computational expense to the info-gap analysis (especially for models having many parameters), evaluating Equations (5) and (6) subject to the constraint imposed by Equation (7) allows the analyst to examine the influence of *individual* uncertain parameters on the robustness or opportunity function. The results of such an exercise can be viewed as analogous to a main-effect sensitivity analysis, where the sensitivity is assessed with respect to the performance function of Equation (3). It is worth repeating that the advantage of estimating sensitivities by way of an IGDT analysis is that there is no reliance on approximating local or global derivatives (or adjoints). Moreover, the IGDT treatment of uncertainty is both general and flexible in that most types of uncertainty (probabilistic or otherwise) can be handled by an IGDT analysis.

Results

1) Data/Model for Fresh Fuel Samples

This discussion begins by considering the effect on

performance (*i.e.* fidelity to data) when all of the four empirically-derived parameters contained in Equation (1) are permitted to vary. Fig. 3 illustrates the robustness curves resulting from IGDT analyses using the info-gap models of uncertainty given by Equations (4) and (7). Table 1 below is provided for clarity.

The primary finding from this analysis is that above approximately $\alpha \cong 15\%$ the overall "worst case performance"[1] of the FRAPCON model (solid, blue line) degrades severely, going from $E_{\mathrm{RMS}}(\boldsymbol{\theta}) \cong 14$ at $\alpha \cong 15\%$ to $E_{\mathrm{RMS}}(\boldsymbol{\theta}) \cong 28$ at $\alpha \cong 25\%$. This degradation in performance appears to be, in large part, tied to the variation of the fourth parameter, which is made more obvious by the "influence curves" presented in Fig. 4.

TABLE 1 PARAMETER LABELING SCHEME FOR SECTION 1)

Parameter Number	Parameter Name
1	A (m-K/W)
2	B (m-K/W/K)
3	E (W-K/m)
4	F (K)

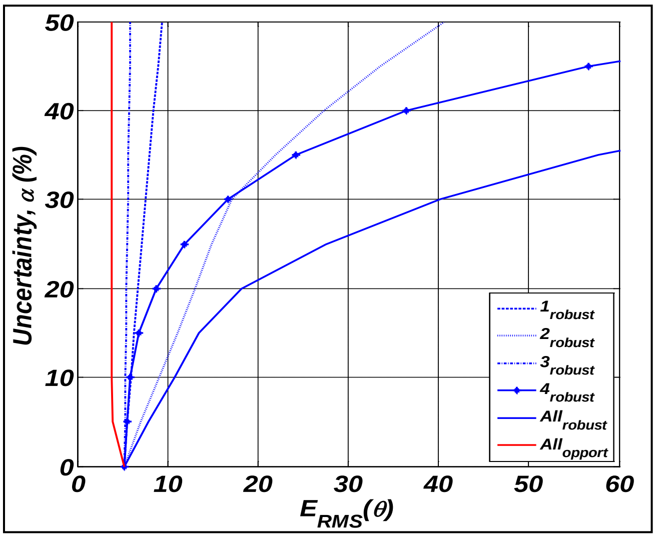

FIG. 3 ROBUSTNESS CURVES FROM THE FOUR-PARAMETER CASE

[1] The authors note that the phrase "worst case performance" is conditioned on a particular value of α. Technically-speaking, a worst case performance does not exist in the context of IGDT, as α is defined simply as an *unbounded* scalar that modulates the space(s) defined by the info-gap models of uncertainty. Practically-speaking, of course, evaluating Equations (5) and (6) requires the definition of α, so the authors use "worst case performance" with this fact in mind.

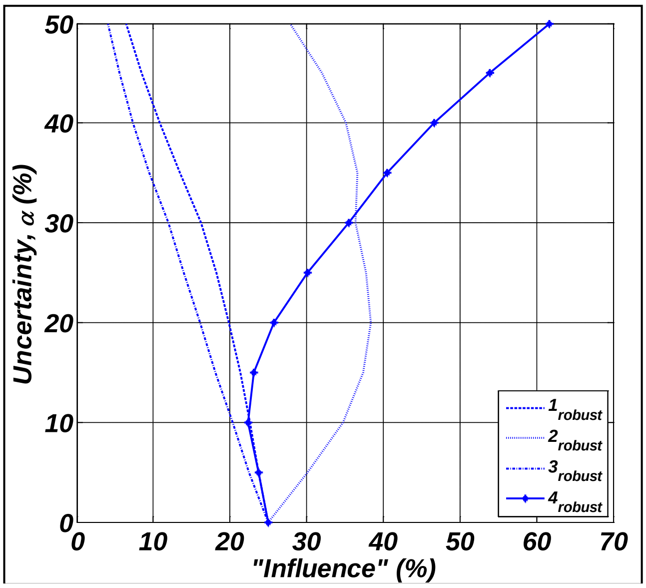

FIG. 4 INLUENCE CURVES FROM THE FOUR-PARAMETER CASE

Influence is computed by normalizing the robustness curves resulting from the IGDT analyses associated with Equation (7). To begin, the inner maximum from Equation (5) is denoted as $\gamma(m_I, \alpha)$, where it is pointed out that this quantity (for various values of α) is, in fact, the quantity plotted in Fig. 3 as the solid, blue line ("All$_{\mathrm{robust}}$"). The analogous inner maximum associated with IGDT analysis of the k^{th} parameter is then denoted as $\gamma_k(m_I, \alpha)$. Finally, a formal statement of influence is given as

$$I(m_I, \alpha) = \frac{\gamma_k(m_I,\alpha)}{\sum_{k=1}^{N} \gamma_k(m_I,\alpha)}, \tag{8}$$

where N is the number of parameters considered in the IGDT analyses associated with Equation (7). Examining Fig. 4, it is seen that at approximately $\alpha \cong 15\%$, the fourth parameter becomes more influential (and the second parameter becomes *less* influential), eventually accounting for approximately 62% of the summation $\sum_{k=1}^{N} \gamma_k(m_I, \alpha)$ at $\alpha = 50\%$. Our earlier analysis indicated that the genetic algorithm optimized (GA-optimized) value of the fourth parameter deviated only slightly from its nominal value (even when the allowable deviation was set at 50%). The results illustrated in Fig. 4 appear to be consistent with this finding, as significant variation of the fourth parameter may lead to poor performance. The converse of this hypothesis also seems likely, as the parameter that varied the farthest from its nominal value in the GA optimization (*i.e.* the third parameter) also exhibits the least influence in Fig. 4.

In light of this knowledge, the authors chose to

execute a similar analysis as the above, except that the fourth parameter is not included in the analysis. Arguably, the most important conclusion from this re-analysis is derived from Fig. 5, as the performance degradation of the FRAPCON model is less dramatic as compared to the four-parameter case. This is not an unexpected result, however, as removing one or more parameters correlated with one or more of the remaining parameters will likely result in improvement of performance (due to the reduced space over which the optimization is performed).

This analysis also represents an example of how the decision-maker could use the results of a previous IGDT analysis to inform a follow-on analysis. In the present case, the authors have asserted that a majority of the performance degradation of the FRAPCON model was due to variation of the fourth parameter, where the obvious action of such a finding would be to reduce or eliminate the allowable variation of the fourth parameter. Having taken such an action, however, the decision-maker may yet be interested in how this alters the overall robustness of the model (*i.e.* "How does the solid, blue line from Fig. 4 change?"). It is expected that the overall robustness of the model would improve, but the degree to which the robustness improves is left to question in the absence of a second IGDT analysis.

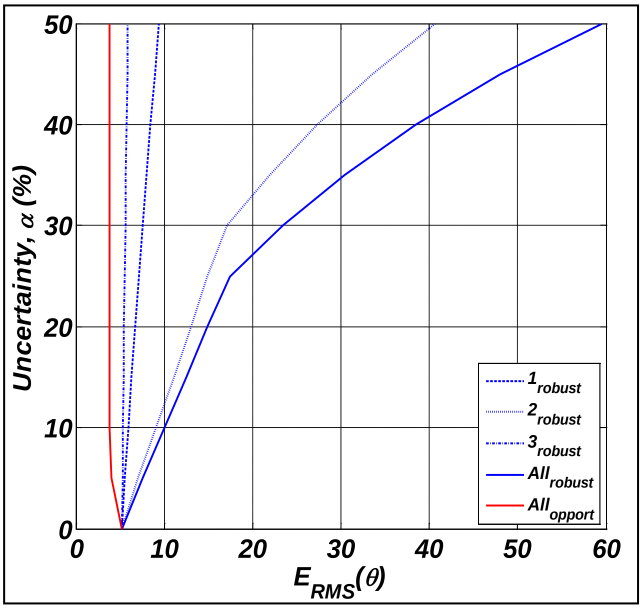

FIG. 5 ROBUSTNESS CURVES FROM THE THREE-PARAMETER CASE

2) *Data / Model for Irradiated Fuel Samples*

Analogously to the previous section, this discussion begins by considering the effect on performance (*i.e.* fidelity to data) when all of the *eleven* empirically-derived parameters contained in Equation (2) are permitted to vary. Fig. 6 illustrates the robustness curves resulting from IGDT analyses using the info-gap models of uncertainty given by Equations (4) and (7). Note that for all of the figures in this section, Table 2 below is provided for clarity.

Fig. 6 and Fig. 7 illustrate similar trends to that observed in the previous section, in that the performance of the FRAPCON model degrades severely as uncertainty is introduced into the IGDT analysis. At first glance, this degradation again appears to follow as a result of varying the fourth parameter, but there are distinguishing characteristics that set this analysis apart from that of the previous section.

TABLE 2 PARAMETER LABELING SCHEME FOR SECTION 2)

Parameter Number	Parameter Name
1	A (m-K/W)
2	B (m-K/W/K)
3	E (W-K/m)
4	F (K)
5	f(Bu)
6	0.9 Factor
7	0.04 Factor
8	g(Bu) A
9	g(Bu) B
10	h(T)
11	Q

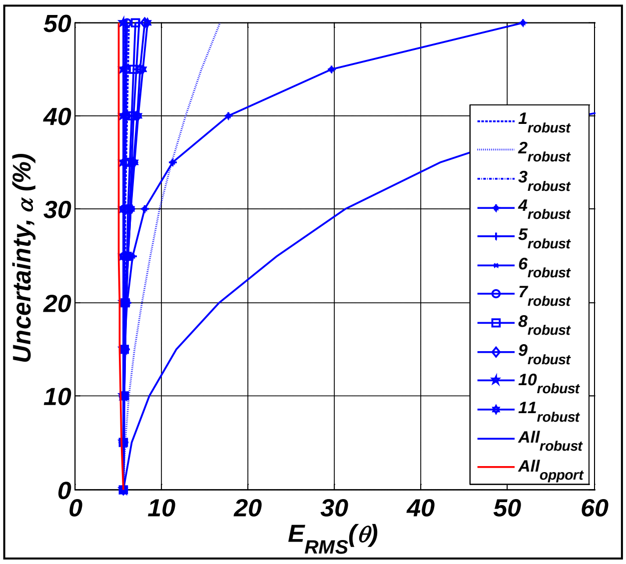

FIG. 6 ROBUSTNESS CURVES FROM THE ELEVEN-PARAMETER CASE

Nevertheless, when $\alpha > 35\%$, the influence of the fourth parameter appears to be significant, and for that reason, the authors again executed an IGDT that did not include the fourth parameter, the results of which are presented in Fig. 8. The primary finding of this analysis is that even in the absence of varying the fourth parameter, the degradation in performance of the FRAPCON model is still severe. Possibly with the exception of the second parameter, such degradation cannot be attributed to a particular parameter. In fact, the robustness curves in Fig. 8 exhibit very high robustness (*i.e.* near negligible sensitivity to parametric uncertainty), yielding performances at $\alpha = 50\%$ that are only a few percent worse than the nominal performance of the model.

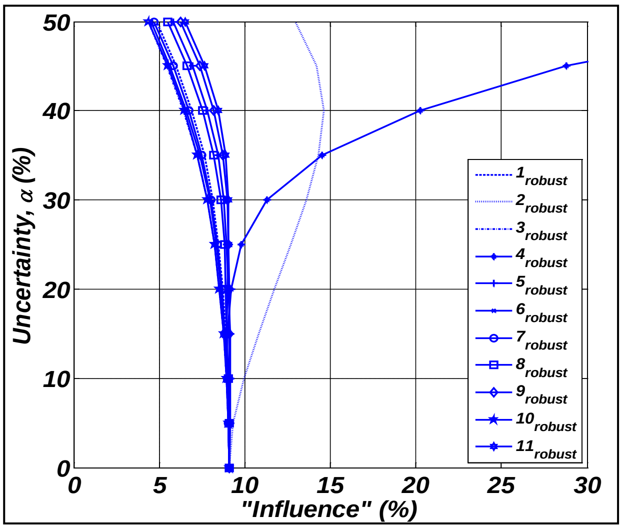

FIG. 7 INFLUENCE CURVES FROM THE ELEVEN-PARAMETER CASE

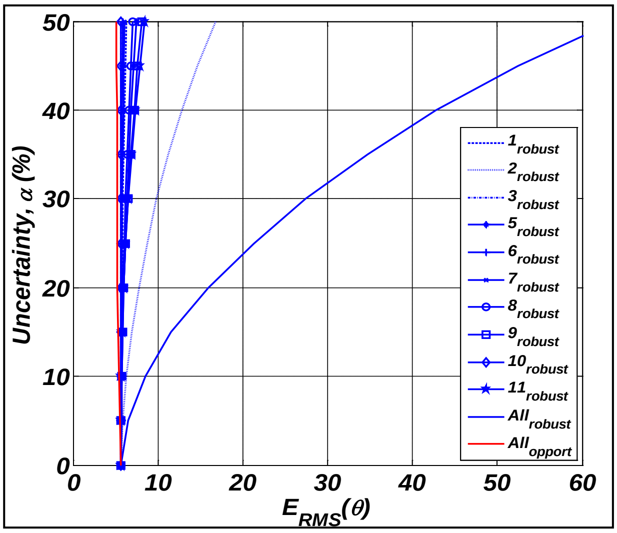

FIG. 8 ROBUSTNESS CURVES FROM THE TEN-PARAMETER CASE

This finding is something of a double-edged sword, as it illustrates that the FRAPCON model is robust to variation in *individual* parameters, but very sensitive to the simultaneous variation of *multiple* parameters. It seems that a reasonable follow-on study would be to consider combinations of parameters to determine whether varying a particular combination of parameters contributes to the majority of the performance degradation reported upon herein.

Conclusion

This paper represents an application of info-gap decision theory to assess the effect of parametric uncertainty in the thermal conductivity model used to make predictions of fuel performance in light water reactors. The predictive capability of the empirical thermal conductivity model adopted by the nuclear fuel performance code, FRAPCON-3.4 is assessed. Other methodologies are applied to the same problem for comparison. The use of MCMC analysis is discussed in the companion paper [Unal, Stull, Williams, 2013]. The genetic algorithm optimization was also used but was not discussed in this paper. While much insight was gained as to the behavior of the models with respect to the available data, the primary conclusion from the genetic algorithm optimization was that the thermal conductivities associated with the fresh fuel samples could *not* be accurately predicted when employing the data/model from the irradiated fuel samples.

This result was confirmed and expounded upon by the second methodology that adopts a Bayesian approach to infer posterior probabilities of the parameters by way of MCMC analyses (see [Unal, Stull, Williams, 2013]). The unique facet of this approach was that it considered the *simultaneous* calibration of the two models to all of the available data, which served to confirm the hypothesis that independent calibration of the models/data should be done with caution, especially when considering the model/data associated with the irradiated fuel samples. The issues associated with this model (*i.e.* Equation (2)) led the authors to question whether or not the model is of the correct mathematical form. Many authors have proposed variants of Equation (2), all of which have fit the available data to one degree or another. Therefore, future efforts will be focused on addressing this issue, possibly by proposing a model that departs from Equation (2), so as to better predict the overall

behavior exhibited by the available data.

The third methodology adopted *Info-Gap Decision Theory* (IGDT) to explore a relatively new concept for quantifying uncertainty of empirically-derived models: *info-gap robustness*. Info-gap robustness, in essence, quantifies the sensitivities of a model to parametric uncertainty, without relying on approximations of global or local derivatives (or adjoints). Furthermore, this uncertainty quantification is performed within the context of a performance metric and yields an intuitive mechanism (*i.e.* the robustness curve) by which analysts may both understand and convey the effects of parametric uncertainties in their model. Generally speaking, the conclusions drawn from this effort are consistent with those of the other two methodologies. However, one of the distinguishing characteristics of the analyses is the unique way in which the results are framed.

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